Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel Level 3 GCE

Paper reference **8FM0/22**

Further Mathematics

Advanced Subsidiary

Further Mathematics options

22: Further Pure Mathematics 2

(Part of option A only)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear.
- Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 5 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

1. Sketch on an Argand diagram the region defined by

$$\underbrace{\left\{z \in \mathbb{C}: -\frac{\pi}{4} < \arg(z+2) < \frac{\pi}{4}\right\}}_{1} \cap \underbrace{\{z \in \mathbb{C}: -1 < \operatorname{Re}(z) \leqslant 1\}}_{2}$$

On your sketch

- shade the part of the diagram that is included in the region
- use solid lines to show the parts of the boundary that are included in the region
- use dashed lines to show the parts of the boundary that are not included in the region

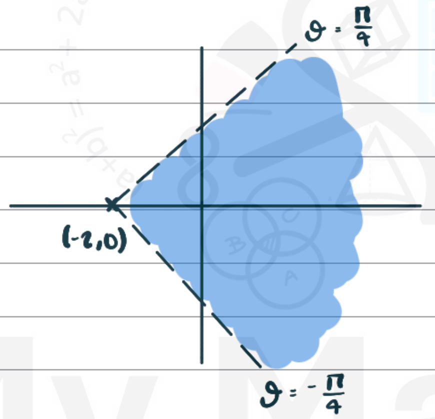
(4)

remember: → if inequality sign is $\geq$ or $\leq$, then use straight lines

→ if inequality sign is $>$ or $<$, then use dotted lines.

1 $\{z \in \mathbb{C}: -\frac{\pi}{4} < \arg(z+2) < \frac{\pi}{4}\}$ → $\{-\frac{\pi}{4} < \arg(z+2)\} \cap \{\arg(z+2) < \frac{\pi}{4}\}$

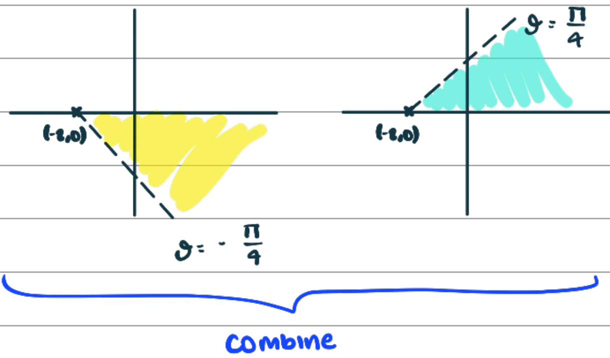
$\{-\frac{\pi}{4} < \arg(z-(-2))\} \cap \{\arg(z-(-2)) < \frac{\pi}{4}\}$



→ must look symmetrical about x-axis

- plot the point (0, -i)
- draw horizontal dotted line from (0,-2)
- draw a line @ $\theta = -\frac{\pi}{4}^c$
- shade region above 0^c

- plot the point (0, -i)
- draw horizontal dotted line from (0,-2)
- draw a line @ $\theta = \frac{\pi}{4}^c$
- shade region below $\frac{\pi}{4}$



P 6 5 5 5 6 A 0 2 2 0

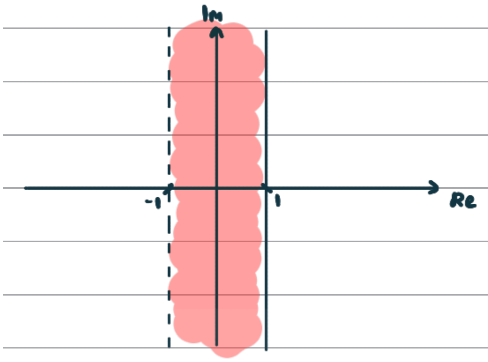
Question 1 continued

② $\{z \in \mathbb{C} : -1 < \mathrm{Re}(z) \leq 1\}$

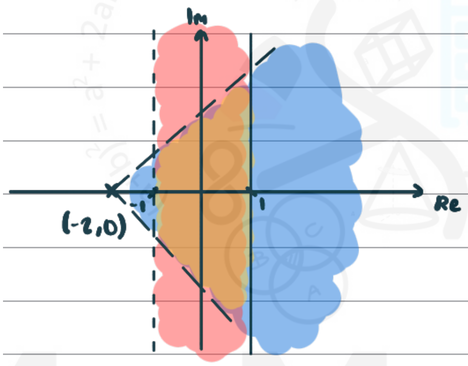
$\{-1 < \mathrm{Re}(z)\} \cap \{\mathrm{Re}(z) \leq 1\}$

- plot the point $(-1,0)$
- draw vertical dotted line from $(-1,0)$
- shade region with x-coord. greater than -1

- plot the point $(-1,0)$
- draw vertical straight line from $(0,1)$
- shade region with x-coord. smaller than 1

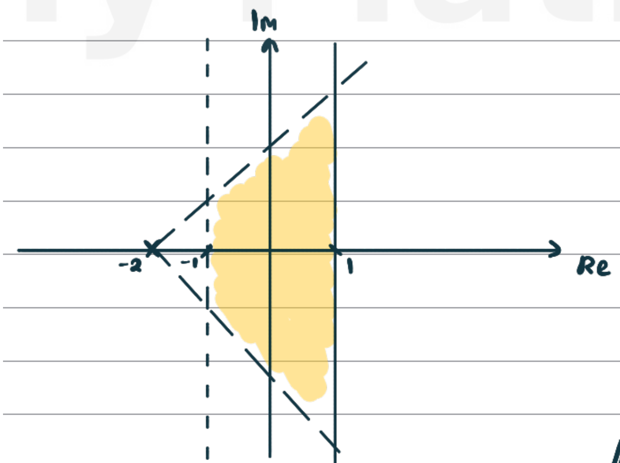


Combine



→ combine both graphs now and look for the overlap between the 2.

represented by yellow shading



//

(Total for Question 1 is 4 marks)

2. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

$$\mathbf{M} = \begin{pmatrix} 4 & 2 \\ 3 & -1 \end{pmatrix}$$

Find a matrix **P** and a diagonal matrix **D** such that

$$\mathbf{P}^{-1}\mathbf{M}\mathbf{P} = \mathbf{D}$$

(7)

$\det(M - \lambda I) = 0$

$M - \lambda I = \begin{pmatrix} 4 & 2 \\ 3 & -1 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} 4 & 2 \\ 3 & -1 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$

$= \begin{pmatrix} 4-\lambda & 2 \\ 3 & -1-\lambda \end{pmatrix}$

$\det(M - \lambda I) = 0$

$\det\left[\begin{pmatrix} 4-\lambda & 2 \\ 3 & -1-\lambda \end{pmatrix}\right] = 0$

$\det(M)$ where $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\det(M) = (a)(d) - (b)(c)$

$(\lambda^2 - 3\lambda - 4) - (6) = 0$

$\lambda^2 - 3\lambda - 4 - 6 = 0$

$\lambda^2 - 3\lambda - 10 = 0$

$(\lambda - 5)(\lambda + 2) = 0$

$\lambda = 5$ or $\lambda = -2$ $\leftarrow$ eigenvalues of M

$D = \begin{pmatrix} 5 & 0 \\ 0 & -2 \end{pmatrix}$ or $\begin{pmatrix} -2 & 0 \\ 0 & 5 \end{pmatrix}$

doesn't matter which order eigenvalues are given in - HOWEVER once you have picked which order, D matrix must be consistent

To work out P, find eigenvectors for each eigenvalue.

DO NOT WRITE IN THIS AREA

Question 2 continued

$M\begin{pmatrix}x\\y\end{pmatrix}=\lambda\begin{pmatrix}x\\y\end{pmatrix}$

$\begin{pmatrix}4&2\\3&-1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=5\begin{pmatrix}x\\y\end{pmatrix}$

$\begin{pmatrix}4x+2y\\3x-y\end{pmatrix}=\begin{pmatrix}5x\\5y\end{pmatrix}$

$4x+2y=5x \quad (1)$

$3x-y=5y \quad (2)$

$(1) \quad 4x+2y=5x$

$2y=x$

when $x=2,\ y=1$

$\therefore$ an eigenvector is $\begin{pmatrix}2\\1\end{pmatrix}$ for the eigenvalue 5

$M\begin{pmatrix}x\\y\end{pmatrix}=\lambda\begin{pmatrix}x\\y\end{pmatrix}$

$\begin{pmatrix}4&2\\3&-1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=-2\begin{pmatrix}x\\y\end{pmatrix}$

$\begin{pmatrix}4x+2y\\3x-y\end{pmatrix}=\begin{pmatrix}-2x\\-2y\end{pmatrix}$

$4x+2y=-2x \quad (1)$

$3x-y=-2y \quad (2)$

$(1) \quad 4x+2y=-2x$

$6x=-2y \quad \div 3$

$3x=-y \quad \times -1$

$-3x=y$

when $x=1,\ y=-3$

$\therefore$ an eigenvector is $\begin{pmatrix}1\\-3\end{pmatrix}$ for the eigenvalue -2

Question 2 continued

$D: \begin{pmatrix} 5 & 0 \\ 0 & -2 \end{pmatrix}$ or $\begin{pmatrix} -2 & 0 \\ 0 & 5 \end{pmatrix}$

$\downarrow$ $\downarrow$ corresponding P n D matrices

$P: \begin{pmatrix} 2 & 1 \\ 1 & -3 \end{pmatrix}$ or $\begin{pmatrix} 1 & 2 \\ -3 & 1 \end{pmatrix}$

$P = \begin{pmatrix} 5 & 0 \\ 0 & -2 \end{pmatrix}$ and $D = \begin{pmatrix} 2 & 1 \\ 1 & -3 \end{pmatrix}$ OR $P = \begin{pmatrix} 2 & 1 \\ 1 & -3 \end{pmatrix}$ and $D = \begin{pmatrix} 5 & 0 \\ 0 & -2 \end{pmatrix}$ //

2.1: The Axioms For A Group | 2.2: Cayley Tables & Finite Groups | 2.3: Order & Subgroups

3. (i) Let G be a group of order 5 291 848

Without performing any division, use proof by contradiction to show that G cannot have a subgroup of order 11

(3)

(ii) (a) Complete the following Cayley table for the set $X = \{2,4,8,14,16,22,26,28\}$ with the operation of multiplication modulo 30 $a * b = a \times b \pmod{30}$

$\times_{30}$	2	4	8	14	16	22	26	28
2	4	8	16	28	2	14	22	26
4	8	16	2	26	4	28	14	22
8	16	2	4	22	8	26	28	14
14	28	26	22	16	14	8	4	2
16	2	4	8	14	16	22	26	28
22	14	28	26	8	22	4	2	16
26	22	14	28	4	26	2	16	8
28	26	22	14	2	28	16	8	4

A copy of this table is given on page 11 if you need to rewrite your Cayley table.

(b) Hence determine whether the set X with the operation of multiplication modulo 30 forms a group.

[You may assume multiplication modulo n is an associative operation.]

(6)

i. assume G has a subgroup of order 11.
By Lagrange's theorem, 11 must divide 5291848.
↳ Must now check if 5291848 is divisible by 11
If an integer is divisible by 11 the alternating sum of its digits is 11.

$5 - 2 + 9 - 1 + 8 - 4 + 8 = 23$
$11 \nmid 23$

Since 23 is not divisible by 11, 11 cannot divide $|G|$
This contradicts Lagrange's theorem

∴ There is no subgroup of order 11. //

DO NOT WRITE IN THIS AREA

Question 3 continued

b. To prove if X is a group, must check the following:

→ closure: all elements in Cayley table are in set X, so closed ✓

→ identity: 16 is the identity ✓

→ inverse:

X	2	4	8	14	16	22	26	28
X^{-1}	8	4	2	14	16	28	26	22

all elements have an inverse ✓

→ associativity: can assume satisfied (as Q states) ✓

All axioms are satisfied, ∴ X is a group under the operation of multiplication modulo 30. //

1.2: The Euclidean Algorithm 1.3: Modular Arithmetic

4. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(i) (a) Use the Euclidean algorithm to find the highest common factor h of 416 and 72 **(3)**

(b) Hence determine integers a and b such that

$$416a + 72b = h$$

(3)

(c) Determine the value c in the set $\{0,1,2...,415\}$ such that

$$23 \times 72 \equiv c \pmod{416}$$

(2)

(ii) Evaluate $5^{10} \pmod{13}$ giving your answer as the smallest positive integer solution. **(3)**

ia. gcd(416,72)

$416 = 5(72) + 56$

$72 = 1(56) + 16$

$56 = 3(16) + 8$

$16 = 2(8)$

$\therefore$ gcd(416,72) = 8

b. $416 = 5(72) + 56 \rightarrow 56 = 416 - 5(72)$

$72 = 1(56) + 16 \rightarrow 16 = 72 - 1(56)$

$56 = 3(16) + 8 \rightarrow 8 = 56 - 3(16)$

$16 = 2(8)$

rewrite part (a) algorithm but now make the remainder the subject.

$8 = 56 - 3\boxed{72 - 1(56)}$ sub in 16 as $\{72-1(56)\}$ from line ②

$8 = 56 - 3(72) + 3(56)$

$8 = 4(56) - 3(72)$

$8 = 4\boxed{416 - 5(72)} - 3(72)$ sub in 56 as $\{416-5(72)\}$ from line ③

$8 = 4(416) - 20(72) - 3(72)$

$8 = 4(416) - 23(72)$

$4(416) - (23)(72) = 8$

$a = 4 \quad b = -23 \quad h = 8$

P 6 5 5 5 6 A 0 1 2 2 0

Question 4 continued

c. $23 \times 72 = 1656$

$1656 \equiv c \bmod (416)$

$a \equiv b \bmod m$ SAME AS $b \equiv a \bmod m$

$c \equiv 1656 \bmod (416)$

$c = 408$

$\boxed{c = 408}$ //

ii. $5^{10} (\bmod 13)$

° Start by calculating smaller powers of 5 mod 13.

$5 \bmod 13 = 5$

$5^2 \bmod 13 = 25 \bmod 13 = 12$ or -1

↳ using this we can say: $5^2 \equiv 12 \bmod 13$ or $\underbrace{5^2 \equiv -1 \bmod 13}$

we will use this going forward as it is easier to work with

$a \equiv b \bmod m$ SAME AS $b \equiv a \bmod m$

$5^2 \equiv -1 \bmod 13$

$5^{10} = (5^2)^5$

raise both sides to the power 5. we know 5^2 and we are trying to find 5^{10}.

$(5^2)^5 \equiv (-1)^5 \bmod 13$

$(-1)^5 = -1$

$\therefore 5^{10} \equiv -1 \bmod 13$

Q asks for smallest positive integer, however.

$-1 \bmod 13 = 12 \bmod 13$

$\boxed{12 \bmod (13)}$ //

4.1: Forming Recurrence Relations 4.2: Solving First-Order Recurrence Relations

5. A person takes a course of a particular vitamin.

Before the course there was none of the vitamin in the person's body.

During the course, vitamin tablets are taken at the same time each day.

u_0

Initially two tablets are taken and on each following day only one tablet is taken.

$10 \times 2 = 20$mg 10mg

Each tablet contains 10 mg of the vitamin.

Between doses the amount of the vitamin in the person's body decreases naturally by 60%

Let u_n mg be the amount of the vitamin in the person's body immediately after a tablet is taken, n days after the initial two tablets were taken.

(a) Explain why u_n satisfies the recurrence relation

$$u_0 = 20 \qquad u_{n+1} = 0.4u_n + 10$$

(2)

The general solution to this recurrence relation has the form $u_n = a(0.4)^n + b$

(b) Determine the value of a and the value of b.

(4)

The course is only effective if the amount of the vitamin in the person's body remains above 6 mg at **all times** throughout the course.

(c) Determine whether this course of the vitamin will be effective for this person, giving a reason for your answer.

(3)

a. Immediately after the first tablets are taken there is 20mg in the person, so $u_0 = 20$

reduction by 60% through day means just before next tablet is taken there is

$0.4\,u_n$ mg of the vitamin left in the person.

The next tablet taken adds 10 mg to the amount just before the tablet is taken,

giving: $u_{n+1} = 0.4u_n + 10 \qquad u_0 = 20$ //

b. $u_n = a(0.4)^n + b$

$u_0 = a(0.4)^0 + b = a(1) + b = a + b$

$u_1 = a(0.4)^1 + b = 0.4a + b$

Form 2 eq^{n}s, then solve using simultaneous to find a and b.

$u_0 = 20$

$u_1 = 0.4\,u_0 + 10$

$= 0.4(20) + 10$

$= 18$

DO NOT WRITE IN THIS AREA

Question 5 continued

$a + b = 20$

$0.4a + b = 18$ $\ominus$

$0.6a = 2$

$a = \frac{2}{0.6} = 10/3$

sub back in to find b

$b = 20 - a$

$b = 20 - (10/3)$

$b = 50/3$

$a = 10/3 \quad b = 50/3$ //

c. must check in the long term first.

$U_n = \frac{10}{3}(0.4)^n + \frac{50}{3}$

as $n \to \infty$, $(0.4)^n \to 0$

$\therefore \frac{50}{3}$ mg vitamin in person's body in the long term.

This is after tablet is taken however - we must check mg before tablet taken.

This is when mg of vitamin in body is lowest - if this is > 6, then is effective.

└ Q states AT ALL TIMES to hint this.

$50/3 - 10 = \frac{20}{3}$ mg ← min. amount of vitamin in body (before tablet is taken)

$\frac{20}{3} > 6$

$\therefore$ there is always at least 6mg of the vitamin in the person - the course of vitamin will be effective //